**Significant Figures:**

When working with measured values, such as length, volume, or mass, scientists need a way to express the degree of certainty (or uncertainty) in their reported numbers. One method for keeping track of measured values is called significant figures.

There are three main questions that can be asked about measured values and results of calculations:

1. Do all numbers of a measurement have meaning?
2. Do all of those numbers in your calculator really count?
3. How can we write numbers so that others will interpret them correctly?

Here is an example of how measurements, and how well we know the actual values, can be problematic. Once I decided to measure the distance between my house and my favorite parking spot at Cuesta.

I measure the distance from my car to my door with a tape measure: 845.3 cm

With my car odometer, from Cuesta to my driveway is: 20.4 km

These two values can’t be added directly, since they have different units, so convert to meters and then add: 8.453 m + 20,400 m = 20,408.453 m

Is this realistic? Is this really the distance between by driveway and my favorite parking spot, to the nearest 0.001 m (nearest 1 mm, or the thickness of a dime)? Of course, measuring a distance of over 12 miles to the nearest thickness of a dime in this manner is silly. The significant figure rules will clear up this silliness, if you remember the rules and the general principles:

- Only report realistic digits in a measurement
- All digits known with certainty + one estimated digit are significant figures.
- In calculations, report results to the fewest sig figs (for multiplication and division) OR fewest decimal places (addition and subtraction).

**Significant Figure Rules** (how to determine which numbers are important)

1. Non-zero digits are always significant.
2. Zeros between non-zero digits are always significant.
   - 1.003 g has 4 Sig Figs
3. Zeros before the first non-zero digit are not significant.
   - 0.0003 g has one SF.
4. Zeros at end of a number with a decimal are significant
   - 450.0 g has 4 SF, 0.0015010 g has 5 SF
5. Zeros at the end of a number w/o a decimal are ambiguous (e.g. 10,300 g). **DON’T USE THEM** Use Scientific Notation instead, to show proper uncertainty: 1.030 × 10^4

**There are reasons for these rules.** Rule 2 with 1.003 g means that the tenths and hundredths places have been measured to be zero. Rule 3 with 0.003 g means that only one digit of the measurement is known in those units, so converting to a different unit, the value would be 3 mg, not 3.00 mg.
OK, so practice the rules and determine how many SF are in each value (mass, in g) below

<table>
<thead>
<tr>
<th></th>
<th># of SF</th>
<th></th>
<th># of SF</th>
<th></th>
<th># of SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.0100</td>
<td>650.0</td>
<td>1.300 × 10²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td>60.02</td>
<td>6050</td>
<td>14.156</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers are posted on the website, check your work now.

**Numbers that end with zero:** What about cases where numbers are being counted, like in 12 eggs or 1,105,000 residents in a city? For the eggs, it’s pretty easy to count 12 whole eggs, with NO room for estimation error. There’s no chance you have 12.1 eggs. The 12 eggs is called an **exact** number, and there is no limitation of significant digits. For the city residents, it would be hard to count all 1 million plus people and be sure of it, as some will move in, some move out, etc. In the case of 1,105,000 residents, the number of significant digits is ambiguous. If one had to use that value in a calculation, it would be best to only work with 4 SF. Numbers with zeros at the end, like 450 m, have an ambiguous number of SF.

For mathematical calculations, there are 2 more rules: one for addition/subtraction and one for multiplication/division:

**For multiplication or division**, use the fewest number of sig figs of any of the numbers in the calculation. This is why you need to know how many SF are there to begin with.

For example, let’s say you measure an object that is 7.6 mm × 8.2 mm, so its area = $62.32 \text{ mm}^2$.
But, these are measurements, with an estimated digit, so the area could be:
$$7.6 \text{ mm} \times 8.3 \text{ mm}, \text{ area} = 63.08 \text{ mm}^2$$

The SF rule is to report to fewest # of SF, so the original calculation should be reported as $7.6 \text{ mm} \times 8.2 \text{ mm}, \text{ area} = 62 \text{ mm}^2$, restricted to 2 SF by either measurement.

What does this do? It tells the reader that the area was only measured to the precision of the nearest 1 mm². The value of 62 mm² means that the actual area could be anywhere between 61-63 mm², and better precision is not known.

**For addition or subtraction**, round off at the number with fewest number of decimal places, ignoring everything you’ve learned about Sig Figs. They don’t matter. For the 2 masses below:

$$658.4 \text{ g} + 0.2209 \text{ g} = 658.6209 \text{ g}$$ if the rules are ignored.

But in this case, one measurement was not nearly as precise, only measuring the nearest 0.1 g. The total can therefore only be $658.6 \text{ g}$, rounding off to the nearest 0.1 g.

So for the distance from my house to my parking spot, the answer should only be 20.4 km, since the odometer doesn’t measure anything past the nearest 0.1 km, or nearest 100 m, and my tape measure at my house didn’t measure anything near 0.1 km.
For combined operations:
If you have a problem with both addition and division, what do you do in terms of the rules? Follow the order of mathematical operations, applying the appropriate Sig Fig rule for that step. The example below shows the intermediate steps, with the proper Sig Figs underlined for each step. There are 3 SF for the multiplication, 1 number past the decimal for the addition, and then 3 SF for the division.

\[(2.756 \times 1.20) \div (9.5 + 11.28) = [3.307 \div 20.78] = 0.159\]

Go ahead and keep as many guard digits (non-significant digits) as you like throughout a calculation, and round off with the correct number of SF at the end.

What do Sig Figs tell us?
What significant figures really do is give some indication of the uncertainty of the measurement. In the distance problem, the measurement of 20.4 km is uncertain to the nearest ±0.1 km, because the odometer only reads to the 0.1 km. For the area problem, the uncertainty is to the nearest ±1 mm². The mass problem says that our combined masses have a value of 658.6 g, ± 0.1 g.

Ratios and Conversions:
For some ratios, there are no limits on significant figures. There are exactly 12 inches in 1 foot, so the ratio of 12 inches/1 foot does not limit a calculation to 1 or even 2 significant figures. All metric conversions are like this too. Multiplying a measured value of 1.567 m by 100 cm/1 m does not limit the answer to 1 or 2 SF, as the answer will be 156.7 cm. Ratios involving measured values do limit sig figs: a density of 0.80 g/1 mL limits the SF in a calculation to 2 Sig Figs.

Scientific notation:
Scientific notation does not affect the Sig Fig rules. If one writes 140.0 or 1.400 \times 10², both values have 4 SF. Scientific notation is a great way to resolve a problem with ambiguous values, like 1400. This value could be written as 1.4 \times 10³, 1.40 \times 10³, or 1.400 \times 10³ to show the proper number of significant digits.

For more information, and a tutorial, see the following web page:

http://www.chem.sc.edu/faculty/morgan/resources/sigfigs/index.html

and a Java based problem set: http://lectureonline.cl.msu.edu/~mmp/applist/sigfig/sig.htm

Complete the following worksheet and turn in at the beginning of class on the due date.
Name: _____________________________ lab time ___________

1. Problems:
How many significant figures are shown in each of the following measured values?

   a. 1432 _________________________  f. 0.00210 __________
   b. 5.91 \times 10^4 ______________________  g. 10.003 _________
   c. 4.00 \times 10^2 ______________________  h. 30.0 __________
   d. 0.309 _________________________  i. 1000 m per 1 km _________
   e. 5000 ______________________  j. 30 (people) __________

Calculate the answers, and write with correct Sig Figs:

   k. 22.37 cm \times 3.10 cm \times 85.75 cm = _______________________
   l. 348.8 g + 14.83 g + 2.1 g = _______________________
   m. (6.54 g -6.34 g) + 1.576 mL = _______________________
   n. (0.0025 cm \times 13.97 cm) + 0.08495 cm^2 = _______________________

2. According to the syllabus, if you miss a quiz because you are sick, when can you make it up?

   a. Whenever I can schedule a time that is convenient
   b. Within a week of when the quiz is given
   c. I cannot make up the quiz, so I will receive a zero for this quiz, which will lower my grade
   d. I cannot make up the quiz, but the lowest quiz will be dropped, so it will not hurt my grade as long as I only miss one quiz

3. According to the syllabus, how many labs can be missed while still earning a C?

   1 2 3 4 it doesn’t matter

4. According to the syllabus, what percentages in the class will earn grades of “C” and :C−”?

   “C” _______________  “C−” ______________________

5. What should you do if you have a question and you just can’t make it to office hours?

6. Is Dr. Baxley scary, or is he genuinely trying to help you succeed in chemistry?