

A **stochastic matrix** (also called a transition matrix) is a matrix whose entries represent the probability that a transition from one state to another state will occur. All of the entries in a stochastic matrix must be real numbers between 0 and 1, inclusive. That is, $0 \leq a_{ij} \leq 1$ for all values of i and j . In addition, all of the entries in each column must add to 1.

Consider the matrix \mathbf{P} , which is a stochastic matrix. Each column represents percentages of voters registered with a particular political party. Each row represents percentages of voters that will align themselves with the indicated party during the next election. The matrix \mathbf{V} gives the initial number of registered voters for each party.

$$\begin{array}{c}
 \text{Currently} \\
 \underbrace{\hspace{2cm}} \\
 \mathbf{P} = \begin{array}{ccc|l}
 \mathbf{R} & \mathbf{D} & \mathbf{I} & \\
 \hline
 0.80 & 0.12 & 0.25 & \text{Republican} \\
 0.15 & 0.86 & 0.15 & \text{Democratic} \\
 0.05 & 0.02 & 0.60 & \text{Independent}
 \end{array} \left. \vphantom{\begin{array}{ccc|l} \hline \\ \\ \\ \end{array}} \right\} \begin{array}{l} \text{Next} \\ \text{election} \end{array}
 \end{array}
 \qquad
 \mathbf{V} = \begin{array}{l|l}
 \begin{bmatrix} 5500 \\ 4100 \\ 400 \end{bmatrix} & \begin{array}{l} \mathbf{R} \\ \mathbf{D} \\ \mathbf{I} \end{array}
 \end{array}$$

For example, the first column in \mathbf{P} tells us that 80% of registered republicans will stay loyal to their party in the next election, but 15% of republicans will switch to the Democratic Party and the remaining 5% will switch to an independent party.

1. What percentage of registered democrats will switch to the Republican Party in the next election?
2. What percentage of independents will stay loyal to their party in the next election?
3. How many voters are initially democrats? What percentage of voters are initially republicans?
4. Find the product $\mathbf{P}\mathbf{V}$ by hand (write it out). What do the entries in $\mathbf{P}\mathbf{V}$ represent?
5. How many registered voters are republicans *after* the first election? What percentage of voters are republicans after the first election?
6. Use your calculator to find $\mathbf{P}^2\mathbf{V}$. Copy it down. What does the first entry in $\mathbf{P}^2\mathbf{V}$ represent?
7. Use your calculator to find \mathbf{P}^2 . Copy it down. What does each entry in the first column represent?

Use your calculator to find \mathbf{P}^{30} and \mathbf{P}^{31} . Notice that the entries in these two matrices are essentially the same. With each additional election, the voting preferences will not change substantially. We say that the population has achieved a *steady state*. Also notice that the values in each row are nearly identical.

8. In the long run, what percentage of registered voters will belong to each of the three parties?