

M229 12:30

QUIZ 5

5.1-5.4

PROVE EACH EQUATION IS AN IDENTITY:

1.
$$\frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$$

2.
$$\cot \theta = \frac{\sin 2\theta}{1 - \cos 2\theta}$$

3. FIND THE EXACT VALUE OF $\cos 105^\circ$.4. SIMPLIFY $1 - 2 \sin^2 75^\circ$ 5. IF $\sin A = \frac{3}{5}$, $A \in \text{QII}$, AND $\sec B = \sqrt{10}$, $B \in \text{QIV}$, FIND:

(a) $\cos(A+B)$

(b) $\sin(2B)$

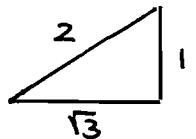
(c) $\tan\left(\frac{A}{2}\right)$

QUIZ 5 KEY

$$\begin{aligned} 1. \frac{\csc \theta - 1}{\cot \theta} &= \frac{(\csc \theta - 1)(\csc \theta + 1)}{\cot \theta (\csc \theta + 1)} \\ &= \frac{\csc^2 \theta - 1}{\cot \theta (\csc \theta + 1)} \\ &= \frac{\cot^2 \theta}{\cot \theta (\csc \theta + 1)} \\ &= \frac{\cot \theta}{\csc \theta + 1} \end{aligned}$$

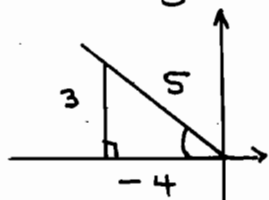
$$\begin{aligned} 2. \frac{\sin 2\theta}{1 - \cos 2\theta} &= \frac{2 \sin \theta \cos \theta}{1 - (1 - 2 \sin^2 \theta)} \\ &= \frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta} \\ &= \frac{\cos \theta}{\sin \theta} \\ &= \cot \theta \end{aligned}$$

$$\begin{aligned} 3. \cos 105^\circ &= \cos(60^\circ + 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2} \left(\frac{1}{\sqrt{2}} \right) - \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{2}} \right) \\ &= \boxed{\frac{1 - \sqrt{3}}{2\sqrt{2}}} \end{aligned}$$

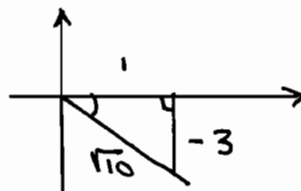


$$\begin{aligned} 4. 1 - 2 \sin^2 75^\circ &= \cos(2 \cdot 75^\circ) \\ &= \cos(150^\circ) \\ &= -\cos 30^\circ \\ &= \boxed{-\frac{\sqrt{3}}{2}} \end{aligned}$$

5. $\sin A = \frac{3}{5}$, QII



$\sec B = \frac{\sqrt{10}}{-1}$, QIV



$$\begin{aligned} \text{(a) } \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= -\frac{4}{5} \left(\frac{1}{\sqrt{10}} \right) - \frac{3}{5} \left(-\frac{3}{\sqrt{10}} \right) \\ &= -\frac{4}{5\sqrt{10}} + \frac{9}{5\sqrt{10}} = \frac{5}{5\sqrt{10}} = \boxed{\frac{1}{\sqrt{10}}} \end{aligned}$$

$$\begin{aligned} \text{(b) } \sin(2B) &= 2 \sin B \cos B \\ &= 2 \left(-\frac{3}{\sqrt{10}} \right) \left(\frac{1}{\sqrt{10}} \right) = \frac{-6}{10} = \boxed{-\frac{3}{5}} \end{aligned}$$

$$\begin{aligned} \text{(c) } \tan\left(\frac{A}{2}\right) &= \frac{1 - \cos A}{\sin A} \\ &= \frac{1 - (-4/5)}{3/5} = \frac{9/5}{3/5} = \boxed{3} \end{aligned}$$