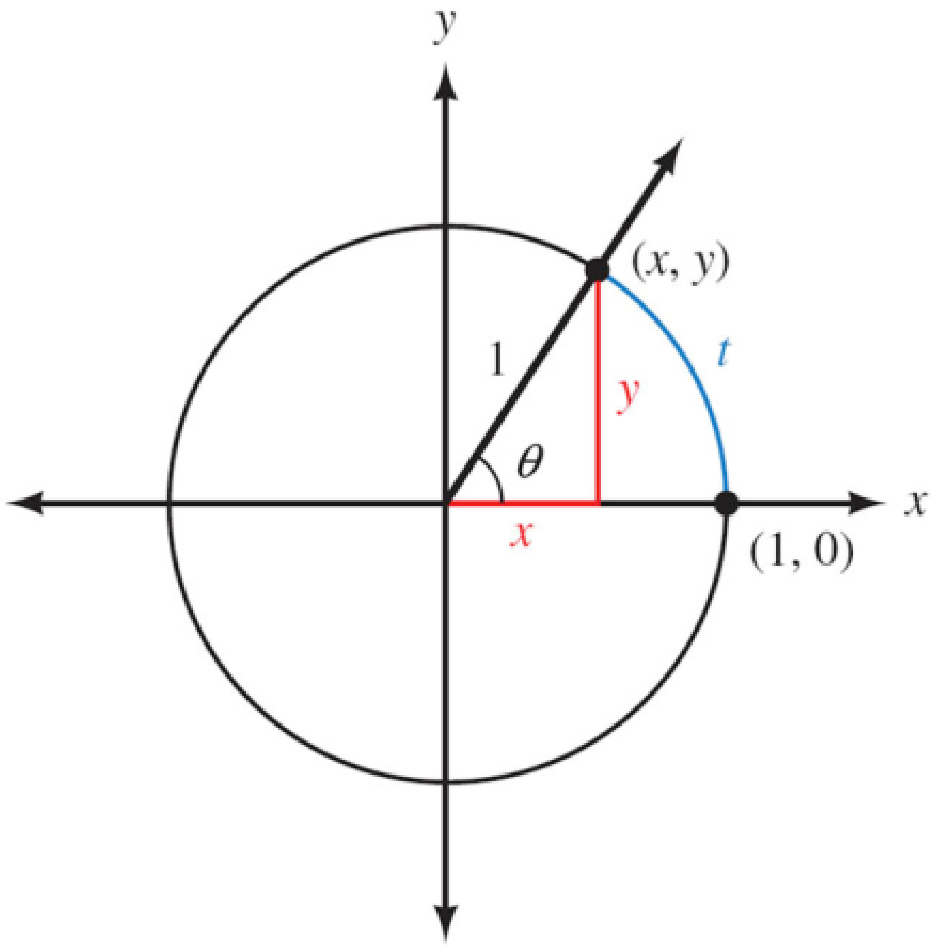
Recall that on the unit circle, $(x, y) = (\cos(t), \sin(t))$ where t is the radian measure of a central angle.



A function is **periodic** if there is some number p such that $f(x + p) = f(x)$ for all x in the domain of f. In other words, the y-values will repeat every p units on the x-axis.

The *smallest positive* value of p is called the **period**.

If a function f has an absolute minimum value $f_{\min}$ and absolute maximum value $f_{\max}$, then we define the **amplitude** of f to be

$$\text{amplitude} = \tfrac{1}{2}(f_{\max} - f_{\min})$$

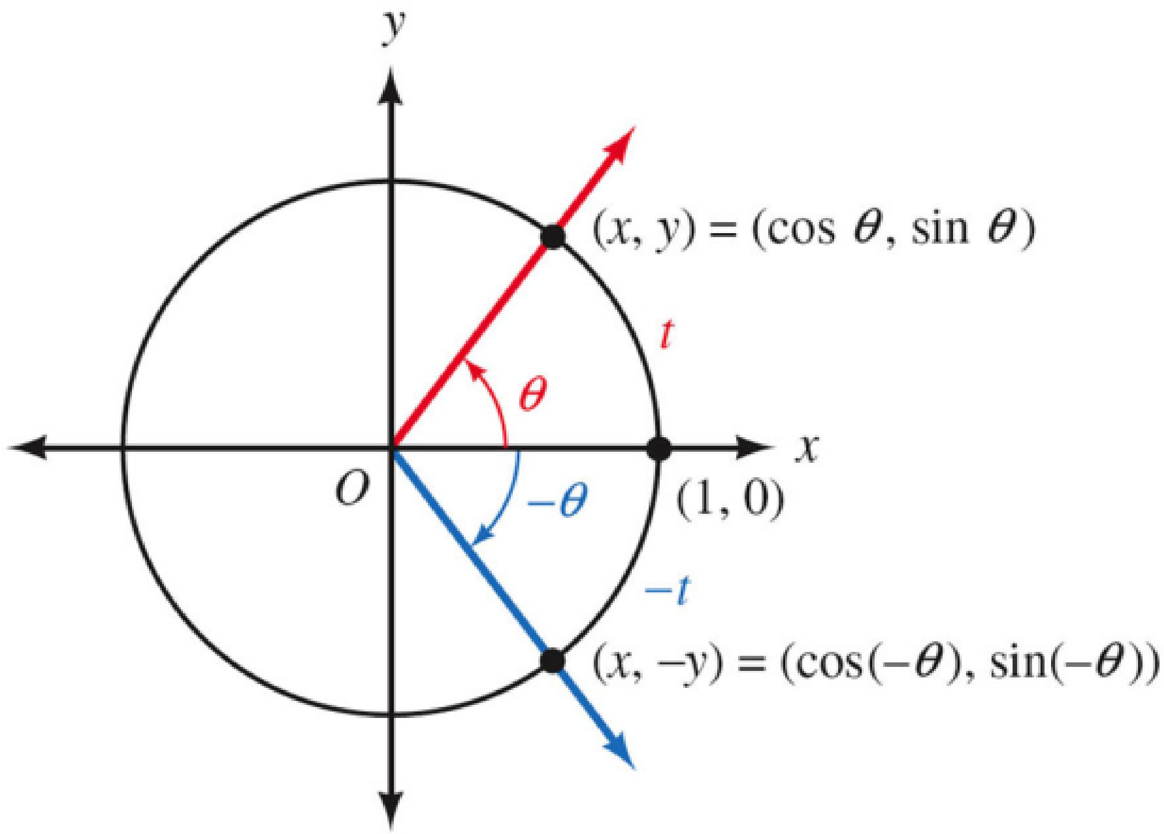
A **zero** of a function $y = f(x)$ is any value $x = c$ from the domain for which $f(c) = 0$.

That is, a zero is an input that results in a zero output.

If a function is symmetric with respect to the y-axis, it is called an **even function**. If it is symmetric with respect to the origin, it is called an **odd function**.

A function $y = f(x)$ is **even** if $f(-x) = f(x)$ for all x in the domain of f. The opposite input yields an *identical* output.

A function $y = f(x)$ is **odd** if $f(-x) = -f(x)$ for all x in the domain of f. The opposite input yields the *opposite* output.


y
(x, y) = (cos θ, sin θ)
t
θ
x
O
−θ
(1, 0)
−t
(x, −y) = (cos(−θ), sin(−θ))

Negative Angle Identities

$\sin(-\theta) = -\sin(\theta)$	Odd
$\cos(-\theta) = \cos(\theta)$	Even
$\tan(-\theta) = -\tan(\theta)$	Odd
$\csc(-\theta) = -\csc(\theta)$	Odd
$\sec(-\theta) = \sec(\theta)$	Even
$\cot(-\theta) = -\cot(\theta)$	Odd

EX: Find the exact value of $\sin\left(-\frac{\pi}{4}\right)$

$$= -\sin\left(\frac{\pi}{4}\right)$$

$$= -\left(\frac{1}{\sqrt{2}}\right)$$

$$= -\frac{1}{\sqrt{2}}$$

EX: Prove the identity $\tan(-\theta)\csc(-\theta) = \sec\theta$

$$\tan(-\theta)\csc(-\theta) = (-\tan\theta)(-\csc\theta)$$

$$= \tan\theta \csc\theta$$

$$= \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$