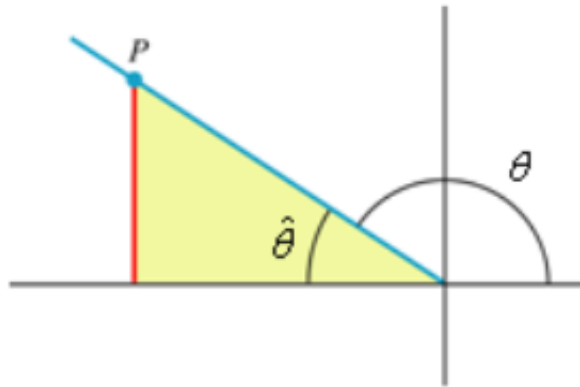


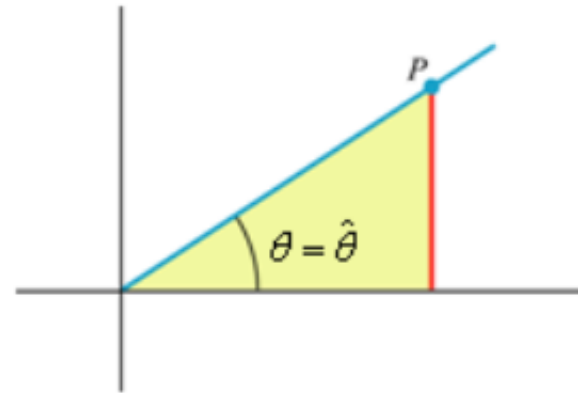
Reference Angles

A *reference triangle* is a right triangle formed by extending a vertical from some point P on the terminal side of θ to the x -axis.

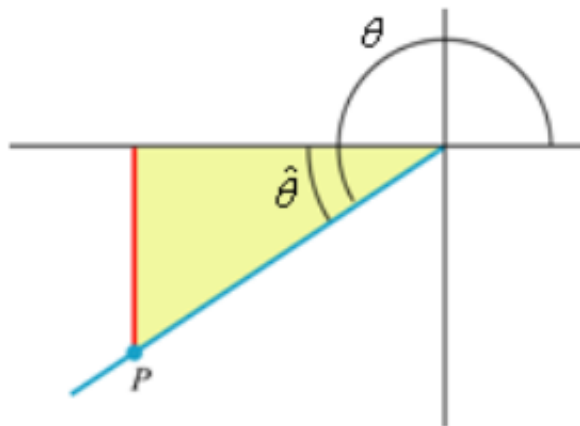
The *reference angle* is the positive acute angle between the terminal side of θ and the x -axis. We will label it $\hat{\theta}$.



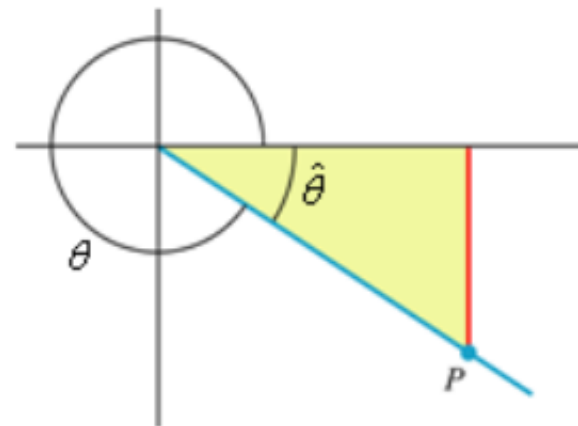
QII: $\hat{\theta} = 180^\circ - \theta$



QI: $\hat{\theta} = \theta$



QIII: $\hat{\theta} = \theta - 180^\circ$



QIV: $\hat{\theta} = 360^\circ - \theta$

To find the value of a trigonometric ratio for any angle θ , find the ratio for its reference angle first, and then determine the correct sign based on the quadrant θ terminates in.

EX: Find $\cos 135^\circ$

$$\theta = 135^\circ \quad \theta \in QII$$

$$\cos 135^\circ = -\cos 45^\circ$$

$$= -\frac{1}{\sqrt{2}}$$

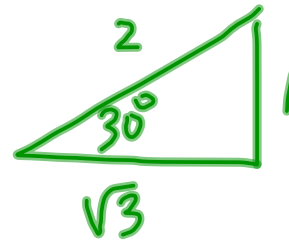
$$\begin{aligned}\hat{\theta} &= 180^\circ - \theta \\ &= 180^\circ - 135^\circ \\ &= 45^\circ\end{aligned}$$

EX: Find $\sin 210^\circ$

$$\theta = 210^\circ \quad \theta \in Q \text{ III}$$

$$\begin{aligned} \sin 210^\circ &= -\sin 30^\circ \\ &= \left(-\frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} \hat{\theta} &= 210^\circ - 180^\circ \\ &= 30^\circ \end{aligned}$$

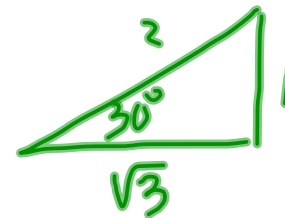


EX: Find $\sec 330^\circ$

$$\theta = 330^\circ \quad \theta \in \text{QIV}$$

$$\begin{aligned} \sec 330^\circ &= + \sec 30^\circ \\ &= \frac{2}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \hat{\theta} &= 360^\circ - 330^\circ \\ &= 30^\circ \end{aligned}$$



Recall that if θ is any angle, then $\theta + 360^\circ k$ for any integer k is coterminal with θ . Coterminal angles have the same trigonometric ratios.

EX: Find $\tan 480^\circ$

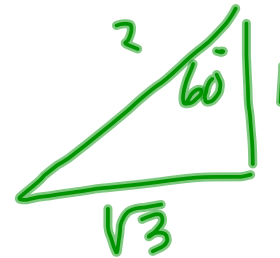
$$480^\circ - 360^\circ = 120^\circ$$

$$\tan 480^\circ = \tan 120^\circ \quad \text{QII}$$

$$= -\tan 60^\circ$$

$$= \boxed{-\sqrt{3}}$$

$$\hat{\theta} = 180^\circ - 120^\circ \\ = 60^\circ$$



EX: Approximate $\sin(-225^\circ)$

JUST USE CALCULATOR!

$$\approx 0.7071$$

EX: Approximate $\sec(590.9^\circ)$

CALC: $\frac{1}{\cos(590.9^\circ)}$
 ≈ -1.5856

To find the angle given the value of one of the trigonometric ratios, we use the appropriate inverse trigonometric function with its reference angle, and then adjust based on the quadrant.

↑
ALWAYS ACUTE
ALWAYS WORK WITH A
POSITIVE VALUE

EX: Find θ if $0^\circ \leq \theta < 360^\circ$, $\cos\theta = -1/2$, and $\theta \in \text{QII}$.

Find $\hat{\theta}$ FIRST

$$\cos \hat{\theta} = \frac{1}{2}$$

MAKE IT POSITIVE

$$\hat{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 60^\circ$$

