Ancient Civilizations

the largest ancient civilizations between 4000 – 1000 BC were

- Egypt; Nile River (1)
- Babylon; Euphrates River (2)
- India; Indus Valley (3)
Prehistoric Mathematics, before 4000 BC:

- Concepts of numbers for counting*
- Concepts of quantity and magnitude for comparison
- Counting implements from 20 to 30 thousand BC
- Some drawings represented geometric objects and designs
- Some artifacts may represent attempts to keep track of time and seasons of the year

*Note:
The development of number systems with names or symbols can only be speculated about since there was no written history from this period, called Prehistory.

People counted using tally marks etched into wood, stone or sand, or other physical representations such as a collection of rocks or sticks. These representations could then be used to compare quantities.

Hand or finger signs representing certain numbers developed over time, possibly leading to some of the written symbols used later in some societies. Basic sounds or words representing numbers may have also developed over this period.
Recorded, or Written, History

begins between 3500 and 3000 BC. The first written system, cuneiform script, developed in Sumer, near Babylon.

The Sumerian numeral system

- base 60
- evolved into the first place-value system, actually a mixed system with grouping of base 10 for numbers smaller than 60
- lacked a symbol for zero until after 300 AD and a radix point (decimal point) so context was required to distinguish some numbers from others

Note: The ancient Chinese also had a place-value or positional number system, rod numerals, by about 450 BC. This also lacked a zero and, like the Babylonians, used a blank column instead. Our current Hindu-Arabic number system originated with the Hindus in India in the first few centuries BC and evolved into a positional number system complete with zero by 400 or 500 AD. It was adopted by the Arabs around 800 AD and transmitted to Europe during Middle Ages.
Mathematics 4000 - 1000 BC that We Know of

Babylonian (Mesopotamian) Mathematics:

- c. 3000 BC: base 60, place-value number system
- c. 2500 BC: Sumerians wrote tablets containing multiplication tables, some geometry problems, and division of whole numbers
- 1800 -1600 BC: tablets from this period contained
  - Fractions
  - Algebra (specific problems and solutions described verbally, using no math symbols or variables)
  - Solutions of linear, quadratic (both by completing the square and using a formula) equations, and some third and fourth degree equations
  - Approximation of $\sqrt{2}$, accurate to 5 decimal places
  - Pythagorean triples (from the Plimpton 322 tablet)
- 1800 – 600 BC: tablets contained
  - Observations of periodic astronomical phenomena
  - Predictions of heliacal risings of planets
  - Mathematics used to predict variation in the lengths of days throughout the year
Mathematics 4000 - 1000 BC that We Know of

Egyptian Mathematics:

- 2000 – 1650 BC: the Rhind Papyrus contained
  - Area formulas
  - Methods for multiplication and division
  - Unit fractions
  - Composite and prime numbers
  - Arithmetic, geometric, and harmonic means
  - Solutions of linear equations
  - Arithmetic and Geometric Series

- 1890 BC: the Moscow Papyrus contained word problems

- 1300 BC: the Berlin Papyrus contained solutions for 2nd degree polynomial equations

- Egyptians had a very detailed understanding of the cycles of stars and planets at least as far back as 3000-4000 BC, and used this to align temples and pyramids with specific stars at specific times of the night and year
Mathematics 4000 - 1000 BC that We Know of

Other Civilizations?

- compared to Babylon and Egypt where the dry climate and writing materials helped preserved ancient written records, very little is known about mathematics in ancient civilizations of Asia and India

- In India, the earlier civilization from 2600 to 1900 BC has left no known documents containing mathematics. Later, after 800 BC, documents were found which contain attempts to square the circle, approximations for $\pi$ and $\sqrt{2}$, a statement of the Pythagorean theorem and a list of Pythagorean triples.

- In China, the first known document containing mathematics was dated to sometime during the first thousand years BC. They had a sophisticated decimal position number system. During the years 300 BC to 200 CE, documents show work on geometry, including some theorems, some physical science, a proof of the Pythagorean theorem, and approximations of $\pi$. 
Mathematics 4000 - 1000 BC that We Know of

The mathematics that developed between 4000 and 1000 BC
- was very practical, or applied (land measurement, interest and tax computation, calendars, etc)
- consisted mainly of verbal descriptions of processes to solve particular problems
- no proofs or demonstrations
- based on observation and inductive reasoning
- little or no abstraction of concepts or connections made between ideas

During the next 1000 years the Greeks
- begin to use deductive reasoning and logic to discover and prove mathematical theorems
- organize the results into a formal mathematical system
Birth of Modern Mathematics, Science, and Philosophy in Ancient Greece

Thales, 600BC:

- known as the first mathematician, scientist, and philosopher because he was first known person to use observation, logic, and deduction to explain natural phenomena rather than mythology
- was popular in Egypt for measuring the heights of the pyramids and the distance from a ship to shore using similar triangles and may have predicted a solar eclipse in 585 BC
- used the Greek word “geometry” which means “earth measurement,” the Egyptian name for mathematics
- named one of the Seven Sages, the seven wisest men in the world

Pythagoras, 550 BC:

- studied in Egypt, was initiated into Egypt’s priesthood and sacred rites, then taken prisoner when the Persians invaded Egypt and studied in Babylon before returning to Greece
- had many mystical beliefs and was attributed with divine powers
- formed the very secretive Pythagorean Society, a group who followed his strict spiritual and mystical beliefs and studied mathematics and numbers as the basis of all reality
- the Pythagorean Society produced the first known proof of the Pythagorean Theorem, the existence of irrational numbers, as well as other mathematical results in geometry and number theory; they are responsible for the term ‘mathematics’
Birth of Modern Mathematics, Science, and Philosophy in Ancient Greece

**Plato, 400 BC:**
- student of *Socrates* (450 BC)
- founded the Platonic Academy where leading mathematicians worked

**Aristotle, 350 BC:**
- student of Plato
- helped develop the foundation of logic
- considered, with Plato and Socrates, as one of the primary founders of Western philosophy

The Great Museum or Library of Alexandria, 330 BC:
- **Alexander the Great**, tutored by Aristotle, built an empire which soon included Egypt where he began building the city of Alexandria
- Alexander was killed shortly after, and one of his generals named Ptolemy (not the mathematician/scientist) became ruler of Egypt and the **Ptolemaic dynasty** ruled there for the next 300 years
- Ptolemy or Ptolemy II finished building Alexandria and founded the famous library there which housed thousands of books (scrolls) from that time and where several of the greatest mathematicians worked including *Euclid, Archimedes* (250 BC), and *Ptolemy* (150 CE, unrelated to the ruler)
- the dynasty comes to an end about 50 BC when Ptolemy XII split his kingdom between his son and daughter, **Cleopatra**, who had a relationship with **Julius Caesar** which kept the Roman Empire from taking over until Caesar was killed shortly after
- the library existed until about 400 AD when the first known woman mathematician and last great scholar to work there, **Hypatia**, was killed by archbishop Cyril and soon after the remaining contents of the library were destroyed
Birth of Modern Mathematics, Science, and Philosophy in Ancient Greece

Euclid, 300BC: little is known about Euclid

- he lived in Alexandria and taught at the great library during the reign of Ptolemy I

- wrote “The Elements”, a work consisting of thirteen volumes, or scrolls, which organized all of the geometric knowledge known by the Greeks at the time, along with some number theory; the first 4 volumes contain what is usually taught in a current geometry course

- responsible for the first formal axiomatic system which consisting of definitions, postulates, theorems, and rigorous proofs based on logic and deductive reasoning which is still in use today
Euclid’s Elements

A rigorous mathematical system consisting of

- definitions of geometric objects
- postulates, or axioms, which are statements given without proof
- theorems proved using definitions, postulates, and other independently prove theorems

Euclid started with 5 basic postulates

- **Postulate 1** A straight line can be drawn from any point to any point.
- **Postulate 2** It is possible to extend a finite straight line indefinitely.
- **Postulate 3** A circle can be drawn with any point as center and any distance as radius.
- **Postulate 4** All right angles are equal.
- **Postulate 5** Given a line \( l \) and a point \( P \) not on \( l \), there is only one line \( m \) containing \( P \) such that is parallel to \( l \).

Note: the 5\textsuperscript{th} postulate is called the “Parallel Postulate”. The version given here is a simpler but equivalent version of Euclid’s. We’ll have more to say about this later.
Euclid’s Elements

Some faults with Euclid’s version

- no undefined terms: causes “circular” definitions and proofs
- unstated assumptions: “obvious” properties or conditions which are not proved nor stated as postulates

Modern axiomatic systems contain improvements due to Hilbert’s around 1900

- undefined terms: terms which obtain their meaning from the axioms only
- axioms or postulates: all assumptions are formally stated
- definitions: terms defined using undefined terms or other definitions
- theorems: statements proved using any of the above and other theorems

References: “Euclid's Window” by Leonard Mlodinow
“An Introduction to the History of Mathematics” by Howard Eves
“Mathematics for the Nonmathematician” by Morris Kline