Polynomials: Summary of Results for Graphs, Zeros, Factors

Behavior (Shape) of Graphs

1. **Leading Coefficient Test** gives the end behavior of the graph

2. **Multiplicity of Zeros** gives the behavior (shape) of the graph at the x-intercepts

3. You can also use a sign chart (+ or -) with test points to help determine graph shape.
   a. end behavior: test a large positive and negative value (eg: x = ±100)
   b. zeros: test a point on either side of each zero

Zeros and Factors

4. **Remainder Theorem**: \( P(c) \) is the remainder when \( P(x) \) is divided by \((x - c)\)

5. **Factor Theorem**: \( P(c) = 0 \) \( \leftrightarrow \) \((x - c)\) is a factor of \( P(x) \)

6. Conclusion from the **Fundamental Theorem of Algebra**: If \( P(x) \) has degree \( n \), then it will have exactly \( n \) complex and/or real zeros counting multiplicity.

7. Complex and Irrational zeros come in conjugate pairs: \( a \pm bi \) or \( a \pm c\sqrt{b} \)

8. Factoring with rational numbers: Every polynomial function can be factored into
   a. linear factors, \( (ax - b) \) or \( x - \frac{b}{a} \) for each rational zero \( \frac{b}{a} \)
   b. *irreducible quadratic factors*, \( (ax^2 + bx + c) \) for each pair of irrational or complex zeros (the quadratic formula will give you the zeros for this factor).

9. Factoring with irrational and complex numbers: Every polynomial function can be factored into \( n \) linear factors of the form \( (x - c) \) counting multiplicity, where \( c \) is a real (rational or irrational) or complex zero.

10. The **Rational Zeros Theorem** gives a list of possible rational zeros for a polynomial and these can be tested using synthetic division and results 4 and 5 above. This is useful when calculators or computers are not being used.

* See Note 2 on the handout “Products of Conjugate Factors”