Section 3.1
Graphing Linear Equations

I. Graphing Ordered Pairs

A. The **Cartesian Coordinate System** is a set of two number lines.

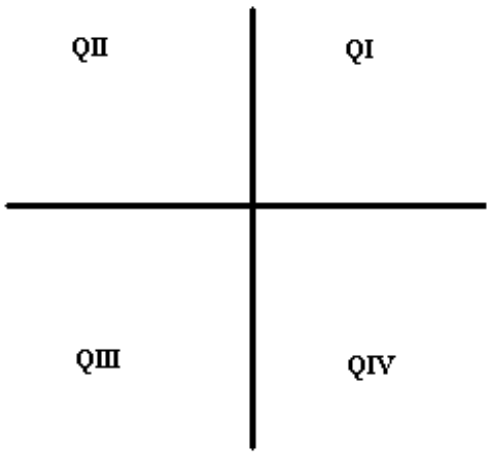
1. The horizontal axis (or number line) is called the **x-axis**.
2. The vertical axis (or number line) is called the **y-axis**.
3. The intersection of the two axes is called the **origin**.

4. An **ordered pair** is a pair of numbers whose order is extremely important that describe the location of a point in the Cartesian Coordinate System – switch the order and you get a different point in the plane.

   a. The first coordinate of the ordered pair is called the **x-coordinate** and tells us the distance that we go horizontally from the origin:
      1. Positive, go to the right
      2. Negative, go to the left

   b. The second coordinate of the ordered pair is called the **y-coordinate** and it tells us the distance that we go vertically from the origin:
      1. Positive, go up
      2. Negative, go down

5. The axes of the Cartesian Coordinate System divide the plane up into four **quadrants** called:
   a. The first quadrant (QI) where x and y are both positive
   b. The second quadrant (QII) where x is negative and y is positive
   c. The third quadrant (QIII) where both x and y are negative
   d. The fourth quadrant (QIV) where x is positive and y is negative



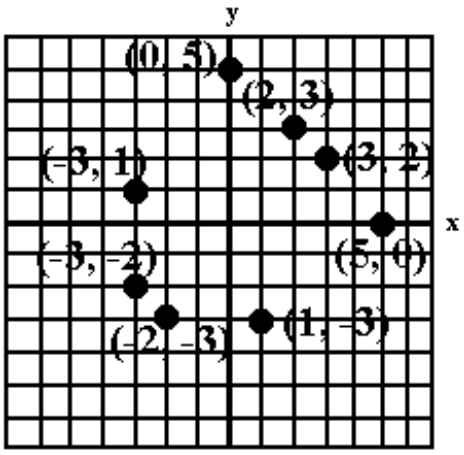
B. To graph an ordered pair, follow the following procedure:

1. Determine the quadrant the point is located in (Note: A point can be located on one of the axes.).
2. From the origin, go
   a. Right if x is positive
   b. Left if x is negative
3. Now go
   a. Up if y is positive
   b. Down if y is negative
4. If either x or y is 0, then your point will be located on one of the axes.

C. Examples – Graph each of the following ordered pairs.

1. (2, 3) 2. (3, 2) 3. (1, -3) 4. (-3, 1)

5. (-2, -3) 6. (-3, -2) 7. (0, 5) 8. (5, 0)



II. Standard Form of a Linear Equation

A. Definitions

1. A **linear equation** is any equation whose graph is a straight line.
2. Any equation whose degree is one is called a **first-degree equation** and its graph is a straight line.
3. An ordered pair is called a **solution point** for a linear equation if substituting for x and y in the equation we get a true statement.
4. A linear equation is in **Standard Form** when it looks like:

$$Ax + By = C$$

where:

a. A, B, and C are integers (i.e. – No fractions)
b. Not both A & B are 0.

B. Testing Solution Points – Check to see if the points are solution points for the linear equation $5x + 4y = 20$

1. (0, 5)

We substitute 0 in for x and 5 in for y in the equation to get:

$5(0) + 4(5) = 0 + 20 = 20$

We get a true statement, so (0, 5) is a solution point.

2. (4, 0)

We substitute 4 in for x and 0 in for y in the equation to get:

$5(4) + 4(0) = 20 + 0 = 20$

We get a true statement, so (4, 0) is a solution point.

3. Now you try one: (-4, 10)

Answer: (-4, 10) is a solution point.

C. A **table of values** is a table that includes ordered pairs that are solution points of an equation.

D. Examples - Complete each table and plot the points.

1. x = 6

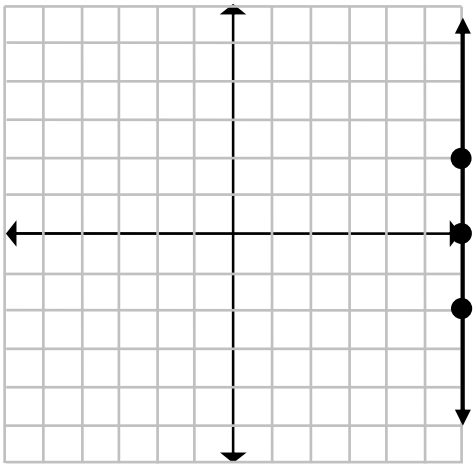
Note that on this problem, we can substitute any value we wish in for y, the value for x must be 6.

x	y
	-2
	0
	2

So to complete this table, we just need to substitute 6 in for x in each row.

x	y
6	-2
6	0
6	2

Now we can plot the points and draw the line.

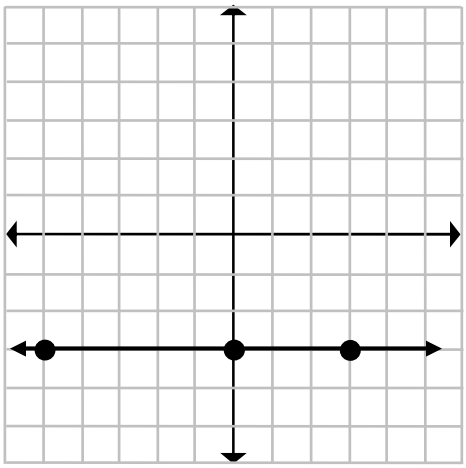


2. Now you try one: y = -3

x	y
-5	
0	
3	

Answer:

x	y
-5	-3
0	-3
3	-3



3. 16x – 8y = 24

x	y
-1	
	-3
	-1

This one will be a little more difficult, we will have to solve three different equations.

First, substitute -1 in for x and solve for y:

16(-1) - 8y = 24

OR

-16 - 8y = 24

Use the Addition Property to isolate the variable term:

-16 + 16 - 8y = 24 + 16

OR

-8y = 40

Use the Multiplication Property to isolate the variable:

$\frac{-8y}{-8} = \frac{40}{-8}$

OR

y = -5

Now we substitute -3 in for y and solve for x:

16x - 8(-3) = 24

OR

16x + 24 = 24

Use the Addition Property to isolate the variable term:

16x + 24 - 24 = 24 - 24

OR

16x = 0

Use the Multiplication Property to isolate the variable:

$\frac{16x}{16} = \frac{0}{16}$

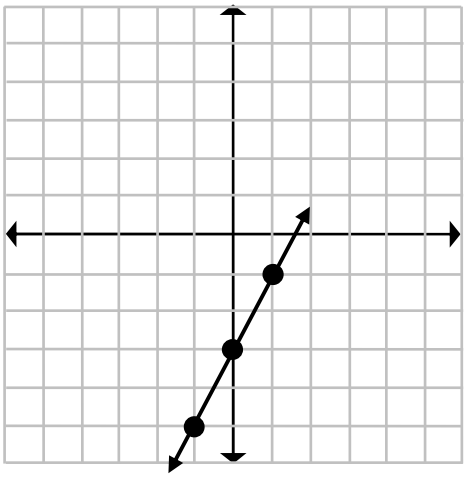
OR

x = 0

Now you substitute -1 in for y and find x.

Answer:

x	y
-1	-5
0	-3
1	-1



E. Applications

1. A football is thrown by a quarterback to a receiver. The points in the figure (middle of left-hand column, page 209) show the height of the football, in feet, above the ground in terms of its distance, in yards, from the quarterback.

   a. Find the coordinates of point B. Then interpret the coordinates in terms of the information given. (Page 209, #88)

   The x-value of point B is 28. The y-value of point B is 7.

   Answer: (28, 7)

   We notice from the graph that the units of x are yards and the units of y are feet. In other words, x tells us how many yards the football is from the quarterback and y tells us how high above the ground the ball is. So interpreting the coordinates of point B, we have:

   Answer: When the football is 28 yards from the quarterback, it is 7 feet above the ground.

   b. What is the football's height when it is caught by the receiver? What is the receiver's distance from the quarterback when he catches the football? (Page 209, #92)

   Remember that the height of the football is y. It appears to me that the football is at a height of 5 feet when it is caught. And the distance that the football is from the quarterback is x. It appears to me that the football (not the receiver!) is 29 yards from the quarterback.

   Answer: The football is 29 yards from the quarterback at a height of 5 feet when it is caught by the receiver.

2. The graph (top of right-hand column, page 209) shows the average number of pounds of chicken and fish that each American consumed in 1960 and 2005.

   a. The graph shows that in 1960, per capita fish consumption was 10 pounds. This increased by approximately 0.15 pound per year from 1960 through 2005. These conditions can be modeled by the mathematical model:

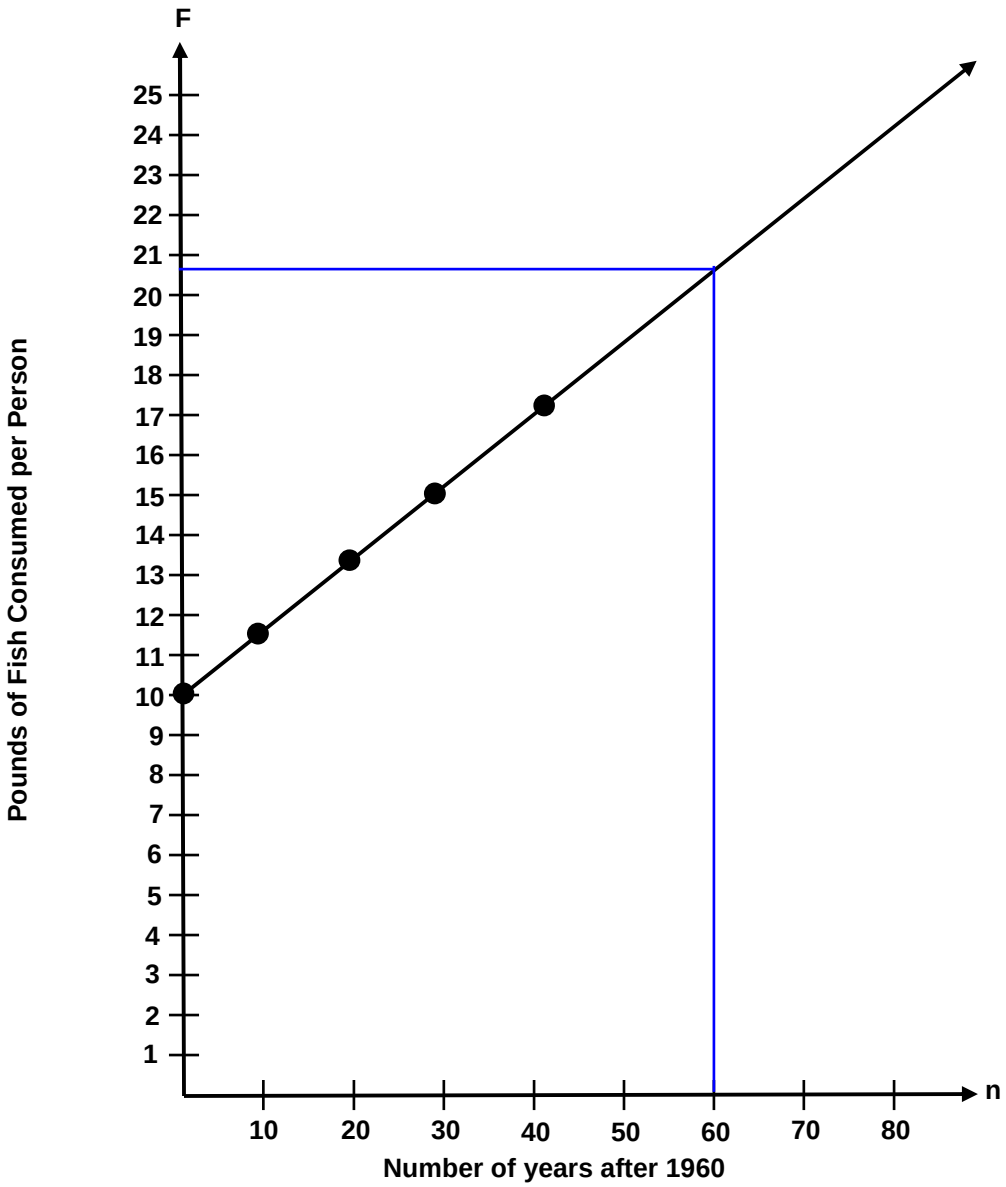
   $$F = 0.15n + 10,$$

   where F is per capita fish consumption n years after 1960. (Page 209, #94)

   1. Let n = 0, 10, 20, 30 and 40. Make a table of values showing five solutions of the equation.

x	y	
0	10	F = 0.15(0) + 10 = 0 + 10 = 10
10	11.5	F = 0.15(10) + 10 = 1.5 + 10 = 11.5
20	13	F = 0.15(20) + 10 = 3 + 10 = 13
30	14.5	F = 0.15(30) + 10 = 4.5 + 10 = 14.5
40	16	F = 0.15(40) + 10 = 6 + 10 = 16

2. Graph the formula in a rectangular coordinate system. Suggestion: Let each tick mark on the horizontal axis, labeled n, represent 10 units. Extend the axis to include n = 80. Let each tick mark on the vertical axis, labeled F, represent 1 unit and extend the axis to include F = 25



3. Use the graph to estimate per capita fish consumption in 2020.

   n = 2020 – 1960 = 60

Answer: By my graph above, per capita consumption will be about 21 pounds by 2020.

4. Use the formula to project per capita fish consumption in 2020.

   F = 0.15(60) + 10 = 9 + 10 = 19

Answer: The formula projects that in 2020, per capita consumption will be about 19 pounds.