RLC Circuits
ELI the ICE man
Series RLC Circuit
Impedance in a series RLC circuit

\[ X_C = \frac{1}{2\pi f C} \]

\[ X_L = 2\pi f L \]

\[ Z = \sqrt{R^2 + X^2} \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]
Series RLC circuit
Current in a series RLC circuit

\[ I = I_R = I_L = I_C \]

\[ I = \frac{V_S}{Z} \]
Voltage in a series RLC circuit

\[ V_R = I \times R \]
\[ V_L = I \times X_L \]
\[ V_C = I \times X_C \]
\[ V_S = \sqrt{V_R^2 + (V_L - V_C)^2} \]
Voltage in a series RLC circuit
Phase angle in a series RLC circuit

\[ \theta = \arctan \left( \frac{V_L - V_C}{V_R} \right) \]

\[ \theta = \arctan \left( \frac{X_L - X_C}{R} \right) \]
Power in a series RLC circuit

\[ P_R = I^2 \times R \]
\[ P_A = V_S \times I \]
\[ PF = \cos \theta = \frac{R}{Z} = \frac{P_R}{P_A} \]

Online Resource for ETCH 213
Faculty: B. Allen
Parallel RLC circuit

Online Resource for ETCH 213
Faculty: B. Allen
Voltage in a parallel RLC circuit

$$V_R = V_L = V_C = V_S$$
Current in a parallel RLC circuit

\[ I_R = \frac{V}{R} \]

\[ I_L = \frac{V}{X_L} \]

\[ I_C = \frac{V}{X_C} \]

\[ I_T = \sqrt{I_R^2 + I_X^2} = \sqrt{I_R^2 + (I_L - I_C)^2} \]
Phase angle in a parallel RLC circuit

$$\theta = \arctan \frac{I_L - I_C}{I_R}$$
Example RLC parallel circuit

\[ V_S = 115 \, \text{V} \]
\[ 60 \, \text{Hz} \]
\[ R = 33 \, \Omega \]
\[ L = 20 \, \text{mH} \]
\[ C = 10 \, \mu \text{F} \]

\[ I_R = \frac{V}{R} = \frac{115 \, \text{V}}{33 \, \Omega} = 3.5 \, \text{A} \]

\[ I_L = \frac{V}{X_L} = \frac{115 \, \text{V}}{7.5 \, \Omega} = 15.3 \, \text{A} \]

\[ I_C = \frac{V}{X_C} = \frac{115 \, \text{V}}{265.3 \, \Omega} = 0.43 \, \text{A} \]

\[ X_L = 2\pi f L = 2\pi \times 60 \times 20 \, \text{mH} = 7.5 \, \Omega \]

\[ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 10 \, \mu \text{F}} = 265.3 \, \Omega \]

Online Resource for ETCH 213
Faculty: B. Allen
Impedance in a parallel RLC circuit

\[ Z = \frac{V_S}{I_T} \]
Power in a parallel RLC circuit

\[ P_R = I_R^2 \times R \]

\[ P_A = V_S \times I_T \]

\[ PF = \cos \theta = \frac{P_R}{P_A} \]
Resonance

Resonance – A circuit condition that occurs when the inductive reactance ($X_L$) is equal to the capacitive reactance ($X_C$).

$F_0 = \text{resonant frequency in hertz}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
Series resonance of an RLC circuit
Quality factor for a series RLC resonance circuit

\[ Q = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{V_C}{V_R} = \frac{X_C}{R} \]
Bandwidth of series resonant RLC circuit

Bandwidth – The width of the group or band of frequencies between the half-power points.

Frequency Response Curve – A graph indicating a circuit’s response to different frequencies.

Cutoff frequency – The frequency at which the gain of the circuit falls below 0.707 of the maximum current or half power (-3dB).
Bandwidth of series resonant RLC circuit (continued)

\[ BW = \frac{f_0}{Q_{f_0}} \]

\[ Q_{f_0} = \text{Quality Factor at } f_0 \]
Series resonant circuit bandwidth

(a) Series resonant circuit diagram

(b) Graph showing current and frequency with resonant frequency at 100 Hz and half-power points at 70.7%
Bandwidth of series resonant RLC circuit (continued)

Half Power Point – A point at which power is 50%. This half power point corresponds to 70.7% of the total current.
Parallel resonance

Parallel Resonance Circuit – A circuit having an inductor and capacitor in parallel with one another, offering a high impedance at the frequency of resonance.
Parallel resonant circuit

\[ V_s \]  

(a)  

No Current  

(b)  

\[ I_s \text{ (Source Current)} = 0; \text{ Therefore Resistance} = \text{Maximum.} \]

\[ (\text{Infinite}) R = \frac{V}{I} = 0 \]

(c)  

Online Resource for ETCH 213  
Faculty: B. Allen
Flywheel action and tank circuit

Flywheel Action – The sustaining effect of oscillation in an LC circuit due to the charging and discharging of the capacitor and the expansion and contraction of the magnetic field around the inductor.

Tank Circuit – A circuit made up of a coil and a capacitor that is capable of storing electric energy.
Quality factor in a parallel resonance

\[ Q = \frac{V_C}{V_S} = \frac{V_L}{V_S} \]
Quality factor in a tank circuit at resonance

\[ Q = \frac{I_{Tank}}{I_S} = \frac{Z_{Tank}}{X_L} \]
Bandwidth in a parallel circuit at resonance

\[ BW = \frac{f_0}{Q f_0} \]
Selectivity

Tuned Circuit – A circuit that can have its components’ values varied so that the circuit responds to one selected frequency, yet heavily attenuates all other frequencies.

Selectivity – The characteristic of a circuit to discriminate between the wanted signal and the unwanted signal.
Applications of RLC circuits

Low-pass filter
High-pass filter
Bandpass filter
Band-stop filter
Low-pass filter
High-pass Filter

Online Resource for ETCH 213
Faculty: B. Allen
Bandpass filter
Band-stop filter
Complex numbers

Complex Numbers – Numbers composed of a real number and an imaginary number

Real Numbers – Numbers that have no imaginary part

Imaginary number – A complex number whose imaginary part is not zero

j Operator – A prefix used to indicate an imaginary number

Complex Plane – A plane whose points are identified by means of complex numbers
Complex plane
Complex numbers examples

- $Y = -4 + j6$
- $W = 3 + j4$
- $Z = -3 - j5$
- $X = 5 - j7$
Coordinates

Rectangular Coordinates – A Cartesian coordinate system whose straight-line axes or coordinate planes are perpendicular.

\[ x + jy \]

Polar Coordinates – Either of two numbers that locate a point in a plane by its distance from a fixed point or a line and the angle this line makes with a fixed line.

\[ r < \theta \]

Online Resource for ETCH 213
Faculty: B. Allen
Representing phasors. (a) Rectangular notation. (b) Polar notation

Online Resource for ETCH 213
Faculty: B. Allen
Polar to rectangular conversion

\[ r < \theta \quad \rightarrow \quad x + jy \]

\[ x = r \cos \theta \]

\[ y = r \sin \theta \]
Rectangular to polar conversion

\[ x + jy \rightarrow r\langle \theta \rangle \]

\[ r = \sqrt{x^2 + y^2} \]

\[ \theta = \arctan \frac{y}{x} \]
Learn to use your calculator to perform

\[ P \rightarrow R \]

and

\[ R \rightarrow P \]

Conversions.
Addition of complex numbers

\[(A + jB) + (C + jD) \]

\[= (A + C) + j(B + D)\]
Subtraction of complex numbers

\[(A + jB) - (C + jD) = (A - C) + j(B - D)\]
Multiplication of complex numbers

\[(A < \theta_1) \times (B < \theta_2)\]

\[= A \times B < (\theta_1 + \theta_2)\]
Division of complex numbers

\[
\frac{A < \theta_1}{B < \theta_2} = \frac{A}{B} < (\theta_1 - \theta_2)
\]
Complex number in ac circuits

On the complex plane:

R is on the Real axis,

$X_L$ is on the $+j$ axis, and

$X_C$ is on the $-j$ axis.
Impedance in a series ac circuit

In rectangular form:

\[ Z_T = R + j(X_L - X_C) \]

In polar form:

\[ Z_T = \sqrt{R^2 + (X_L - X_C)^2} \left< \arctan\left( \frac{X_L - X_C}{R} \right) \right] \]
Current

\[ I = \frac{V_S}{Z_T} \]

Where \( V_S \) and \( Z_T \) are in polar form
End of RLC Circuits